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AN APPLICATION OF THE BRANCH-AND-BOUND METHOD TO THE CATALOGUE ORDERING PROBLEM

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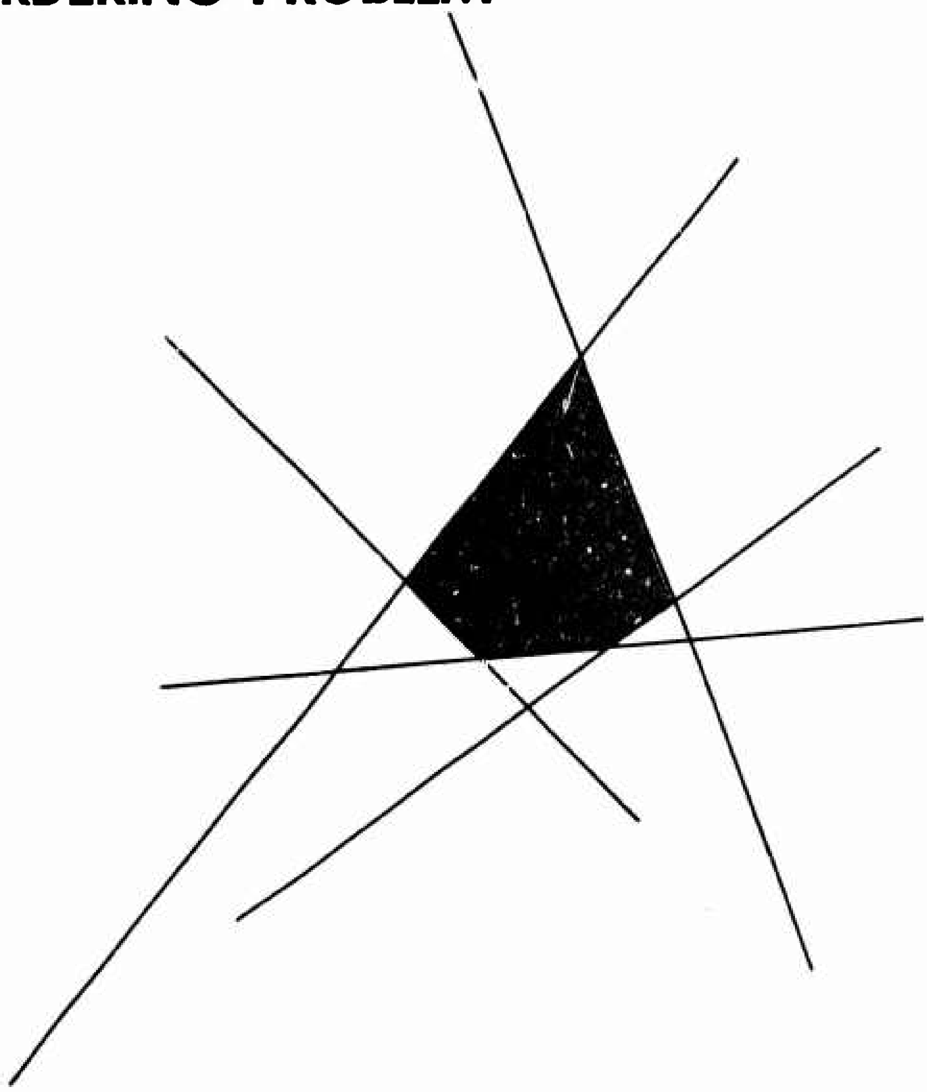

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1. INTRODUCTION

A parallel (or series) system of components is considered. Each component of the system is assumed to be chosen from a set of available components each with a specified reliability and a specified cost. The problem, that of minimizing the cost of the system while guaranteeing a specified system reliability, can be written as a zero-one integer program. A branch-and-bound algorithm is the suggested solution method.

2. STATEMENT OF THE PROBLEM

Consider a parallel† coherent system in n components. Let the i-th component, c_i, come from a set, S_i, of possible components. That is, $c_i \varepsilon S_i, i = 1,2, \ldots, n$. Assume $S_i \cap S_j = \phi$, for $i \neq j$. Let k_i = number of elements in S_i. Associated with each element s_{ij} of S_i is a reliability p_{ij} and a cost c_{ij}.

We would like to choose c_1 from S_1, c_2 from S_2, etc., in such a way as to minimize the cost of constructing the system while guaranteeing a system reliability of at least α.

We will assume that in each set S_i, there is no pair of elements s_{ij} and s_{ik} such that $p_{ij} > p_{ik}$ and $c_{ij} \leq c_{ik}$, because if there were, we would never consider element s_{ik}. Therefore, $p_{ij} > p_{ik} \Longleftrightarrow c_{ij} > c_{ik}$.

†The analysis is the same if the system is in series.

Let $x_{ij} = \begin{cases} 1 & \text{if } s_{ij} \varepsilon S_i \text{ is chosen for the } i\text{-th component of the system} \\ 0 & \text{otherwise.} \end{cases}$

The system reliability can be written

$$R = 1 - \prod_{i=1}^{n} \prod_{j=1}^{k_i} (1 - p_{ij})^{x_{ij}} . \tag{1}$$

The constraint $R \geq \alpha$ is equivalent to $\sum_{i=1}^{n} \sum_{j=1}^{k_i} a_{ij}x_{ij} \leq K$ where $a_{ij} = \ln(1 - p_{ij})$ and $K = \ln(1 - \alpha)$.†

3. THE MATHEMATICAL PROBLEM

The problem can be stated mathematically as follows:

$$\begin{aligned} &\text{Minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{k_i} c_{ij}x_{ij} \\ &\text{Subject to} \quad \sum_{i=1}^{n} \sum_{j=1}^{k_i} a_{ij}x_{ij} \leq K \\ &\qquad \sum_{j=1}^{k_i} x_{ij} = 1 \qquad i = 1,2, \ldots, n \\ &\qquad x_{ij} = 0 \text{ or } 1 \qquad \text{for all } i,j \end{aligned} \tag{I}$$

Problem (I) is a linear integer programming problem. The method used to solve this problem starts by solving a linear program which is identical to Problem (I) with the exception that the constraint $x_{ij} = 0$ or 1 for all i,j is replaced by $x_{ij} \geq 0$ for all i,j Call the LP problem Problem (II).

†Note that for the system in series, the form of the constraint is the same with $a_{ij} = -\ln p_{ij}$ and $K = -\ln \alpha$.

Problem (II) has $\sum_{i=1}^{n} k_i$ variables and $n + 1$ constraints. A simple check for the feasibility of Problem (II) is as follows: Let j_i = second index of $\min \{a_{i1}, a_{i2}, \ldots, a_{ik_i}\}, i = 1,2, \ldots, n$. Let $x_{1j_1} = x_{2j_2} = \ldots = x_{nj_n} = 1$ and all other $x_{ij} = 0$. Then if this solution is not feasible, no integer solution will be feasible and we will terminate. If this solution is feasible, there will be an LP solution. Since the variables of Problem (II) are all bounded between zero and one, we know it has an optimal solution and the optimal functional value z_o is a lower bound to the solution of Problem (I).

Let us say we use the simplex method on Problem (II) and arrive at an optimal solution $\underset{\sim}{X}^*$.

Since Problem (II) has $n + 1$ constraints, at optimality at most $n + 1$ components of $\underset{\sim}{X}^*$ will be positive and all others are zero.

Let us call $x_{i1}, x_{i2}, \ldots, x_{ik_i}$ the i-th group of variables, $i = 1,2, \ldots, n$.

<u>Theorem 1</u>:

At optimality of Problem (II), one of the following two conditions obtains:

(1) In $n - 1$ of the n groups of variables, exactly one variable equals one and all others are zero. In the n-th group, two of the variables are positive and their sum is one.

(2) In all n groups of variables, exactly one variable equals one and all others are zero. The slack variable in the inequality constraint will either be positive or it will be zero but basic.

<u>Proof</u>:

Suppose in each of two groups, there were two positive variables. Since there are at most $n + 1$ positive variables, there would be at most

$n + 1 - 4 = n - 3$ positive variables remaining. But there would be remaining $n - 2$ constraints of the form

$$\sum_{j=1}^{k_i} x_{ij} = 1 . \tag{2}$$

Since this constraint would be impossible to meet in $n - 2$ groups with only $n-3$ positive variables, we must conclude that at most one group will have two positive variables. A similar argument shows that no single group could have three positive variables. This being the case, $n - 1$ groups have one positive variable each. Since each of these groups has the constraint (2), that one positive variable in each of these $n - 1$ groups must equal one. The n-th group then has either one or two positive variables (it cannot have no positive variables because it, too, must satisfy (2)). If it has two positive variables, (2) tells us their sum is one. If it has one positive variable, (2) says it must equal one, and the $(n + 1)$st variable can only be the slack in the inequality. The slack variable must then be basic in the optimal solution, though it could still have zero value. //

If condition (2) above obtains, we have an integer solution and we are finished. Let us assume that condition (1) obtains. That is, we have solved Problem (II) and have obtained a solution in which one group has two positive variables and all others have one positive variable. For convenience, let us reorder the variables so that the first one in each group is positive. In the group with two positive variables, the variables are reordered so that it is the first two variables which are positive. Finally, the rows are reordered so that it is the first group that contains two positive variables. Figure 1 shows the tableau at optimality of Problem (II).

$$
\begin{array}{lllllll}
 & & \bar{c}_{13}x_{13}+\ldots+\bar{c}_{1k_1} & +\bar{c}_{22}x_{22}+\ldots+\bar{c}_{2k_2}x_{2k_2} & +\ldots\ldots+ & \bar{c}_{n2}x_{n2}+\ldots+\bar{c}_{nk_n}x_{nk_n}+\bar{g}w-z & =-z_o \\
\boxed{x_{11}} & & +\bar{a}_{13}x_{13}+\ldots+\bar{a}_{1k_1} & +\bar{a}_{22}x_{22}+\ldots+\bar{a}_{2k_2}x_{2k_2} & +\ldots\ldots+ & \bar{a}_{n2}x_{n2}+\ldots+\bar{a}_{nk_n}x_{nk_n}+\bar{d}w & =1-t \\
 & \boxed{x_{12}} & +\bar{b}_{13}x_{13}+\ldots+\bar{b}_{1k_1} & +\bar{b}_{22}x_{22}+\ldots+\bar{b}_{2k_2}x_{2k_2} & +\ldots\ldots+ & \bar{b}_{n2}x_{n2}+\ldots+\bar{b}_{nk_n}x_{nk_n}+\bar{f}w & =t \\
 & & & \boxed{x_{21}}+x_{22}+\ldots+x_{2k_2} & & & =1 \\
 & & & & \ddots & & \\
 & & & & & \boxed{x_{n1}}+x_{n2}+\ldots+x_{nk_n} & =1
\end{array}
$$

FIGURE 1: TABLEAU AT OPTIMALITY IN PROBLEM (II)

where the variable "w" is the slack variable. Those variables in boxes are the basic canonical variables which form the LP solution. It can be shown that

$$\bar{c}_{ij} = c_{ij} - \left\{ c_{i1} + (c_{12} - c_{11}) \frac{a_{ij} - a_{i1}}{a_{12} - a_{11}} \right\} \tag{3}$$

$$\bar{g} = - \frac{c_{12} - c_{11}}{a_{12} - a_{11}} \; ; \; \bar{d} = \frac{-1}{a_{12} - a_{11}} \; ; \; \bar{f} = \frac{1}{a_{12} - a_{11}} = -\bar{d} \tag{4}$$

$$\bar{a}_{1j} = \frac{a_{12} - a_{1j}}{a_{12} - a_{11}} \; ; \; \bar{a}_{ij} = \frac{a_{i1} - a_{ij}}{a_{12} - a_{11}} \; i = 2,3, \ldots, n \tag{5-a}$$

$$\bar{b}_{1j} = 1 - \bar{a}_{1j} \tag{5-b}$$

$$\bar{b}_{ij} = -\bar{a}_{ij} \qquad i = 2,3, \ldots, n \tag{5-c}$$

$$t = \frac{K - \sum_{i=1}^{n} a_{i1}}{a_{12} - a_{11}} \; ; \; z_o = \sum_{i=1}^{n} c_{i1} + (c_{12} - c_{11})t \; . \tag{6}$$

Since the solution is optimal, we know that $0 \leq t \leq 1$, $\bar{c}_{ij} \geq 0$, $i = 1,2, \ldots, n$, $j = 1,2, \ldots, k_i$, and $\bar{g} \geq 0$.

Now we introduce the method of "Branch-and-Bound"† to the problem. As the tableau in Figure 1 indicates, $x_{11} = 1 - t$, $x_{12} = t$, and $x_{i1} = 1$ for $i = 2,3, \ldots, n$. All other variables are zero. We will partition the node at the origin into three nodes as indicated in Figure 2.

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†See [1].

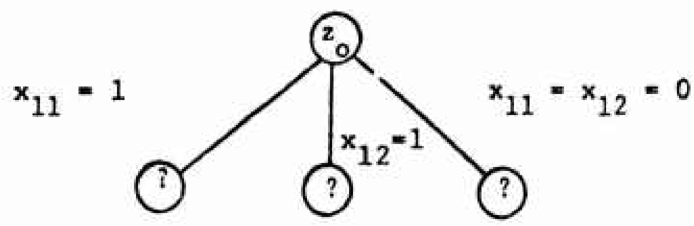


FIGURE 2: BRANCHING FROM THE ORIGIN

where z_o in the origin node indicates the lower bound to all integer solutions.

We now solve Problem (II) with the added constraint $x_{11} = 1$. This problem is an LP problem in $\sum_{i=2}^{n} k_i$ variables and n constraints. The initial tableau can be written by inspecting the tableau of Figure 1. It is given in Figure 3.

$$
\begin{array}{lll}
\text{cost row} & \bar{c}_{22}x_{22}+\dots+\bar{c}_{2k_2}x_{2k_2} + \dots + \bar{c}_{n2}x_{n2}+\dots+\bar{c}_{nk_n}x_{nk_n} + \bar{g}v - z & = -z_o \\
\text{1-st row} & \bar{b}_{22}x_{22}+\dots+\bar{b}_{2k_2}x_{2k_2} + \dots + \bar{b}_{n2}x_{n2}+\dots+\bar{b}_{nk_n}x_{nk_n} + \bar{f}w & = t \\
\text{2-nd row} & \boxed{x_{21}} + x_{22}+\dots+ x_{2k_2} & = 1 \\
\vdots & \ddots & \vdots \\
\text{n-th row} & \boxed{x_{n1}} + x_{n2}+\dots+ x_{nk_n} & = 1
\end{array}
$$

FIGURE 3: INITIAL TABLEAU FOR PROBLEM (II) WITH ADDED CONSTRAINT $x_{11} = 1$

This tableau is not in canonical form because there are only $n - 1$ basic variables; namely, $x_{21} = x_{31} = \dots = x_{n1} = 1$.

Before we proceed any further, however, we check (in the same manner as before) to see if our problem is feasible with the added constraint $x_{11} = 1$. If it is not feasible, we prune this branch. We proceed assuming it to be feasible.

The n-th basic variable is obtained by pivoting on one of the coefficients in the first row of the set of n constraints. We will pick this element so as to maintain the nonnegativity of the cost coefficients. If there is no positive $\bar{b}_{1j}$, $\bar{f}$ must be positive. We pivot on $\bar{f}$ making the slack variable, w, basic. If there is at least one positive $\bar{b}_{1j}$, then we find the indices "rs" which satisfy

$$\frac{\bar{c}_{rs}}{\bar{b}_{rs}} = \min\left\{ \frac{\bar{c}_{1j}}{\bar{b}_{1j}} \;\middle|\; \bar{b}_{1j} > 0 \right\} \tag{7}$$

If $\bar{f} > 0$ and $\frac{\bar{g}}{\bar{f}} \leq \frac{\bar{c}_{rs}}{\bar{b}_{rs}}$, it is again the case that the slack variable, w, should be added to the basis. In this case, the solution to the LP problem in Figure 3 is integer. We will not partition any further from this node; and in Figure 2, the question mark at the end of the arc marked "$x_{11} = 1$" is replaced by the optimal functional value of this LP problem. This would be our first integer feasible solution to Problem (I). From this point on, we would accept integer solutions only if the corresponding functional value is less than that of the current integer solution. We will denote the best feasible integer solution by $\underset{\sim}{Y}^*$ and the functional value by y^*.

If, on the other hand, $\bar{f} < 0$ or $\bar{f} > 0$ but $\frac{\bar{g}}{\bar{f}} > \frac{\bar{c}_{rs}}{\bar{b}_{rs}}$, then we pivot on element $\bar{b}_{rs}$. In this case, we find that the basic canonical solution is

$$x_{21} = x_{31} = \dots = x_{r-1,1} = 1 \; ; \; x_{r1} = 1 - \frac{t}{\bar{b}_{rs}} \, , \; x_{rs} = \frac{t}{\bar{b}_{rs}} \; ; \; x_{r+1,1} =$$
$$\dots = x_{n1} = 1 \, , \text{ and all other } x_{ij} = 0 \, .$$

The question now is, "Is $x_{rs} \leq 1$?" If it is, we immediately have the

solution to this constrained LP problem. If it is not, then we are dual feasible but not primal feasible.

We now apply the dual simplex method to solve this problem. Since we start with only one negative element on the RHS, we can expect that the solution, on the average, should come quickly. When the solution is obtained, the functional value z_1 is put in place of the question mark in Figure 2.

If the LP solution is integer, then, we need not partition that node further; and we say that we have a feasible integer solution $\underset{\sim}{Y}^*$ and its corresponding functional value y^* .

If the LP solution is noninteger, then when we partition further from this node, we will partition on the two noninteger variables as we did after the original LP problem (as shown in Figure 2).

To solve the problem at the node along the arc marked "$x_{12} = 1$" , we must first reconstruct the optimal tableau of Figure 1. This is facilitated by storing in memory the indices of the basic variables in the optimal solution to the first LP problem. From this point on, the problem is handled identically to the way it was handled along the arc marked "$x_{11} = 1$" . We will call the optimal value of the functional of this problem z_2 (assuming it exists).

Along the arc marked "$x_{11} = x_{12} = 0$" we have a slight change. First we check to see if the problem is feasible in the same manner as mentioned earlier. Assuming it is, we reconstruct the tableau of Figure 1 and remove the variables x_{11} and x_{12} (they are zero at any successor node to this node). Now the tableau is not in canonical form. To put it in canonical form, we will pivot on two elements, one at a time. With each pivot, we will remain dual feasible. In the first row of the constraints, if none of the $\bar{a}_{1j}$ is positive, then $\bar{d} > 0$ must hold. We pivot on $\bar{d}$. If, however, at least one of the $\bar{a}_{1j}$ is positive, we

find the indices "rs" which satisfy

$$\frac{\bar{c}_{rs}}{\bar{a}_{rs}} = \min\left\{ \frac{\bar{c}_{ij}}{\bar{a}_{ij}} \;\middle|\; \bar{a}_{ij} > 0 \right\}. \tag{8}$$

Having computed (8), if $\bar{d} < 0$, we pivot on $\bar{a}_{rs}$. If, on the other hand, $\bar{d} > 0$, we pivot on $\bar{a}_{rs}$ if $\frac{\bar{c}_{rs}}{\bar{a}_{rs}} < \frac{\bar{g}}{\bar{d}}$. If the inequality goes the other way we pivot on $\bar{d}$.

Since we are assuming the problem is feasible, we must have at least one of the variables positive in the first group. Thus, if the pivot element that we just found is in any group other than the first, on the second pivot we must choose an element from the first group and must maintain the nonnegativity of the cost coefficients with that pivot. We will see shortly that this is always possible.

If this first pivot element is *not* from the first group, then after pivoting on it, the new tableau will appear schematically (i.e., the line indicates the existence of nonzero elements) as in Figure 4.

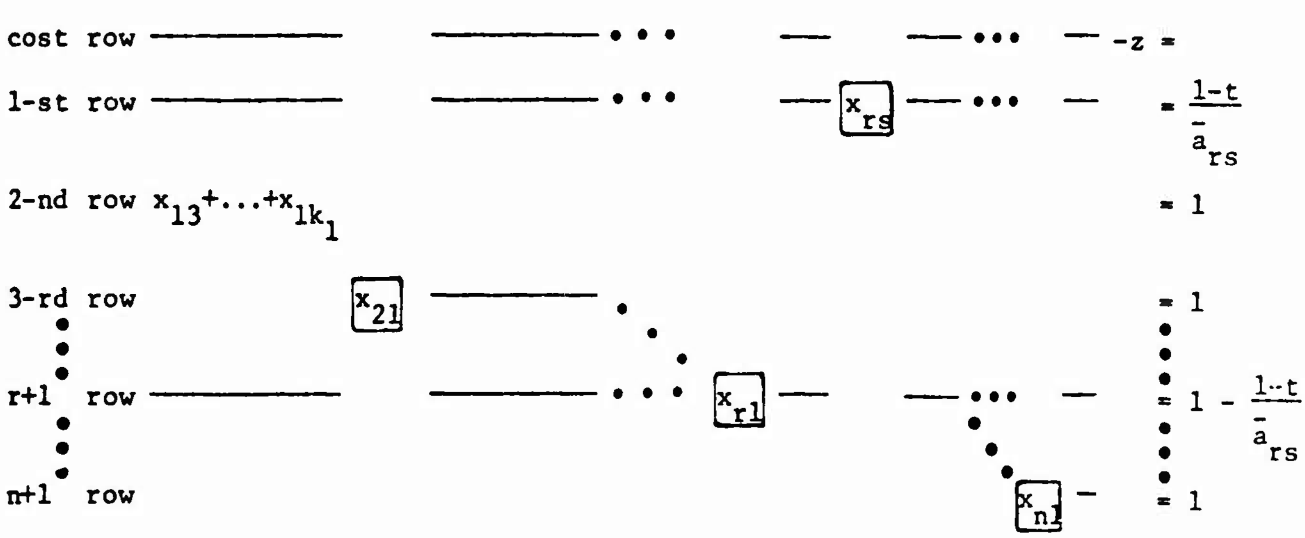


FIGURE 4: SCHEMATIC REPRESENTATION AFTER PIVOTING IN FIRST ROW

Notice that by pivoting on $\bar{a}_{rs}$, the entire second row vanishes, except for the elements of the first group in which the new coefficients are all one. That this is true follows from Equations (5-b) and (5-c).

The second pivot will be on an element in the second row. Since all coefficients not in the first group have vanished, the pivot can only be on an element in the first group.

So as to maintain the nonnegativity of the cost coefficients, the second pivot element will be chosen by finding the index "q" such that

$$\bar{\bar{c}}_{1q} = \min \left|\bar{\bar{c}}_{1j}\right| . \tag{9}$$

Note that a double bar has been used to indicate that, because of our first pivot on $\bar{a}_{rs}$, these coefficients are no longer those in Figure 1. The schematic representation of the canonical tableau is shown in Figure 5.

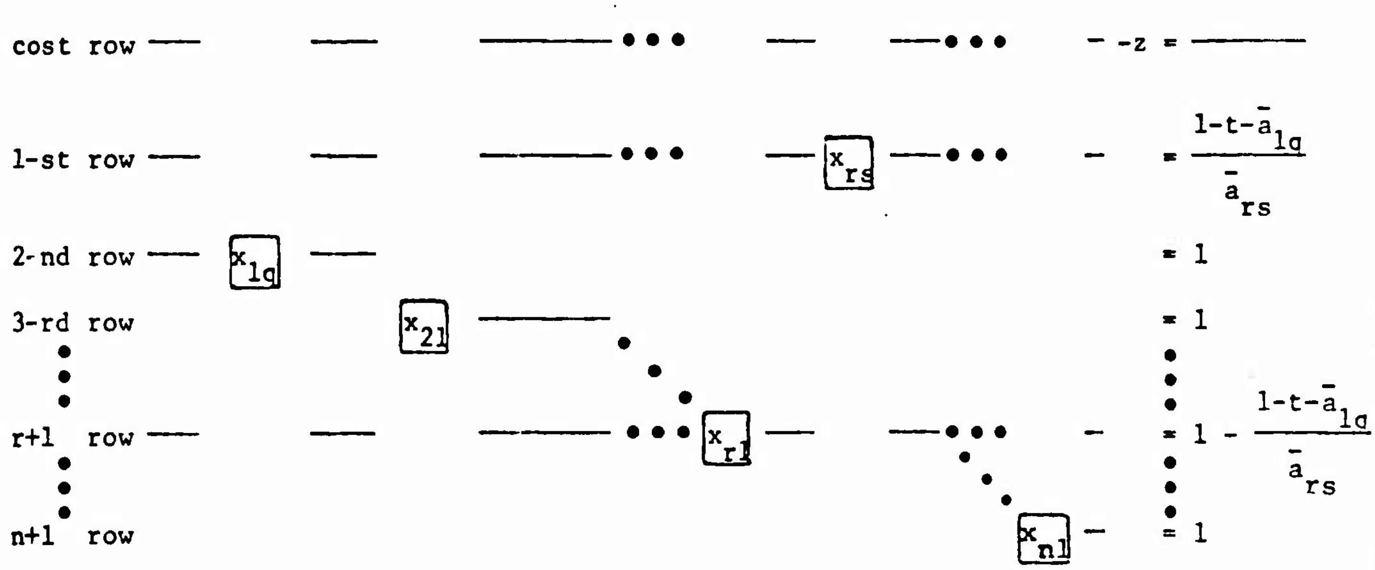


FIGURE 5: SCHEMATIC REPRESENTATION AFTER THE SECOND PIVOT

Since we chose the two pivot elements so as to maintain the nonnegativity of the cost coefficients, we know we are dual feasible. We will be primal feasible if and only if

$$0 \leq \frac{1-t-\bar{a}_{1q}}{\bar{a}_{rs}} \leq 1 \,. \tag{10}$$

If (10) holds, then we are at optimality. If not, we apply the dual simplex method which will yield the optimum solution and functional value. If at optimality the slack variable, w, is basic, we have an integer solution and we need not partition this node any further. The optimal functional value is compared with any previous integer solution functional value to see if we will keep the current solution or discard it because we already have a better one. If at optimality, we have a noninteger solution, the functional value, z_3, replaces the question mark in Figure 2 and when we partition this node further, the partitioning will be on the two noninteger variables as we did after the original LP problem.

At this point, we have concluded the first step of the problem. Figures 6 and 7 show two possible trees at the end of the first step.

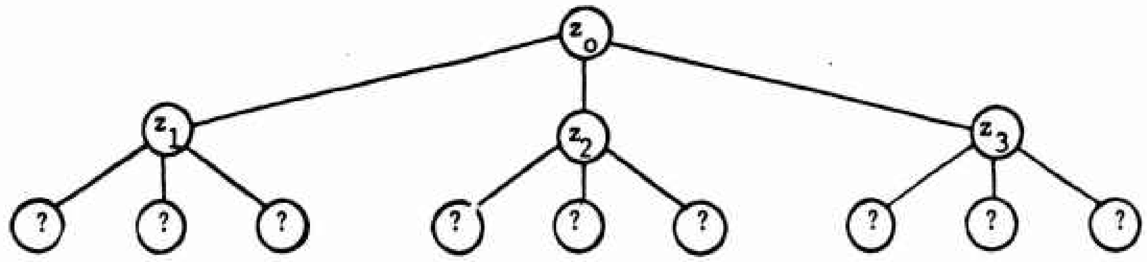


FIGURE 6: A POSSIBLE TREE AFTER THE FIRST STEP

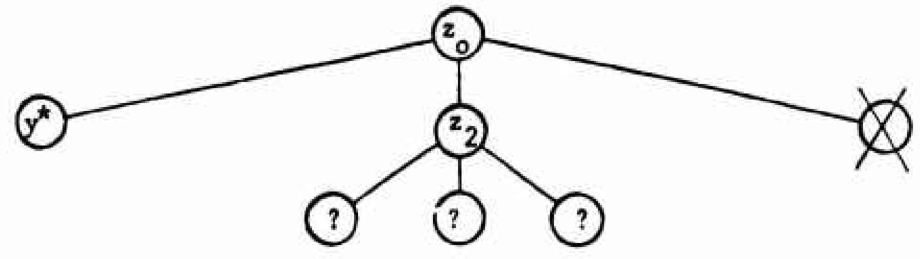


FIGURE 7: ANOTHER POSSIBLE TREE AFTER THE FIRST STEP

where y^* indicates an integer solution was found when we constrained $x_{11} = 1$, and the crossed-out node indicates that no feasible solution could be found when we constrained $x_{11} = x_{12} = 0$.

The second step of the problem begins by first choosing from all feasible pendant nodes, the minimum of the z_i (we say "feasible" because, as in Figure 7, some nodes may not need to be partitioned further), and we will carry on the problem from this point exactly as if we were starting at the beginning. That is, we solve three new LP problems in exactly the same way we solved the last three.

At the m-th step of the problem, the tree may appear as in Figure 8.

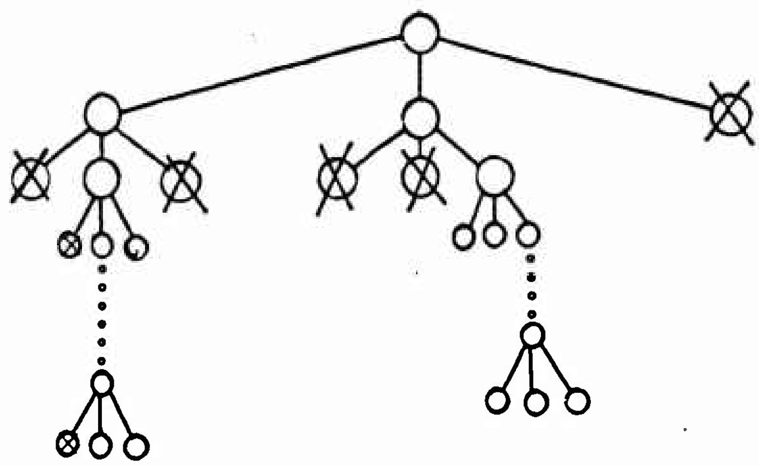


FIGURE 8: A POSSIBLE TREE AT THE m-th STEP

The darkened node is our best integer solution so far, and the clear pendant nodes have yet to be dealt with.

Hopefully, as we progress further down the tree, more and more nodes will appear that do not need to be partitioned further. There are two main ways in which this can happen: (1) if an integer solution is found, and (2) if the node becomes infeasible--this can happen in one of three ways:

(a) The constrained LP problem at that node is infeasible

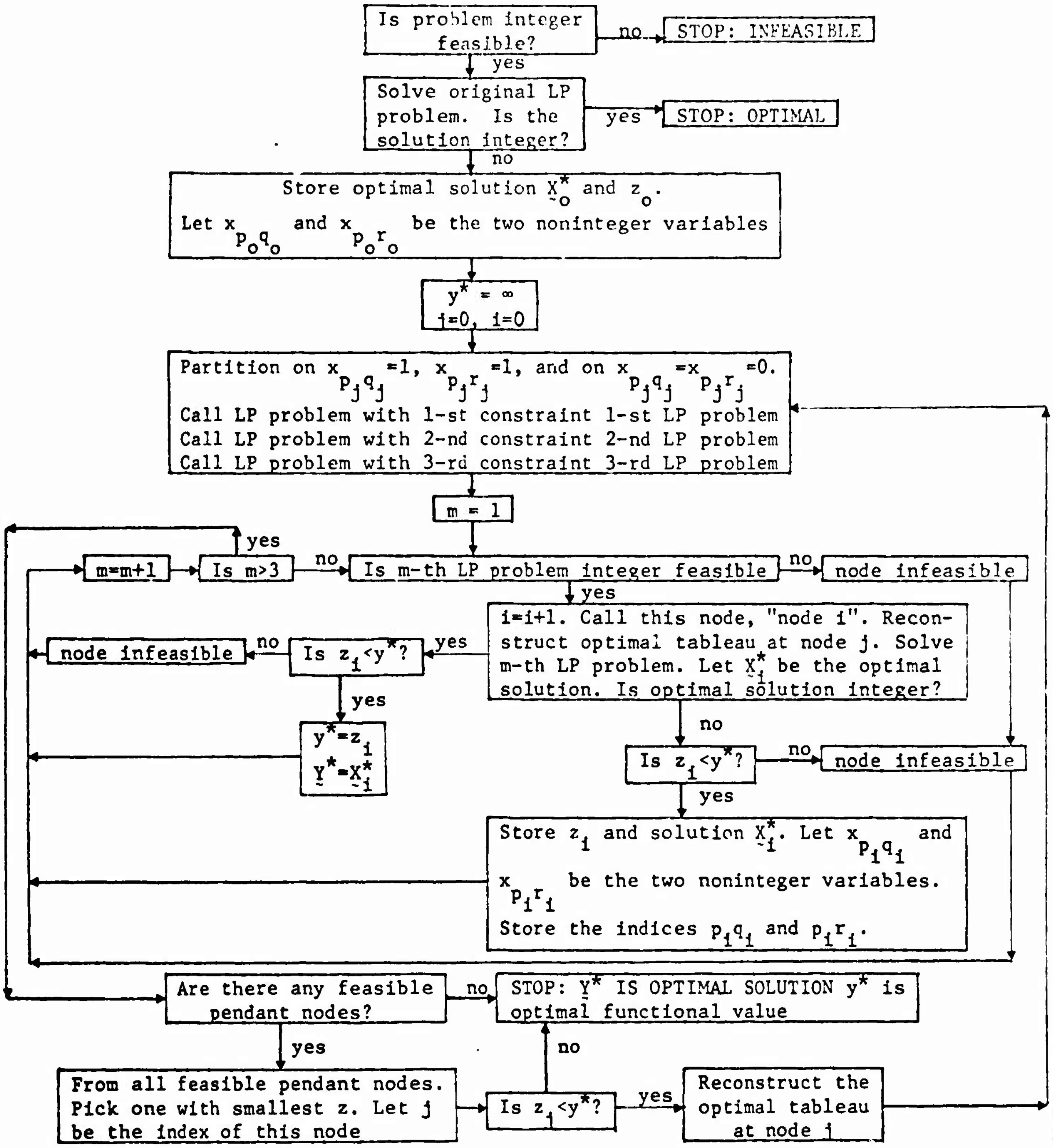
(b) The constrained LP problem at that node is feasible, but the optimal solution (possibly integer) yields an optimal functional value which is not less than the current best integer solution functional value

(c) A new integer solution is found which has a functional value which is smaller than the LP functional value at a previous node (we will then say that the *previous* node is infeasible).

The algorithm will terminate when all feasible pendant nodes have LP optimal functional values no less than the current best integer solution functional value.

A flow diagram of the algorithm is given in Figure 9.

FIGURE 9: FLOW DIAGRAM OF THE ALGORITHM



4. COMMENT

At this time, the algorithm has not been programmed; nor is it anticipated that it will in the near future. This being the case, no statement can be made of its efficiency.

REFERENCE

[1] Lawler, E. L. and D. E. Wood, "Branch-and-Bound Methods: A Survey," Operations Research, Vol. 14, pp. 699-719, (1966).

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There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.

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